Liouville's theorem (complex analysis)

Intro

Statement

In complex analysis, Liouville theorem states that every bounded entire function must be constant. That is, every holomorphic function for which there exists a positive number

such that for all belongs to .

Proof

Method 1:

With fact of holomorphic functions are analytics,

If is an entire function, it can be represented by Tylor series about 0.

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where

= =

And denotes the circle about 0 of radius .

Suppose is bounded. We can estimate directly.

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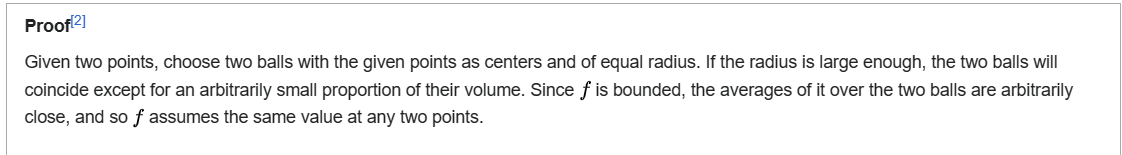
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Therefore, letting tends to inf, will gives one that = 0 for all . Thus,

= =which completes the proof.

Method 2:



Ref

[Liouville's theorem (complex analysis) - Wikipedia](https://en.wikipedia.org/wiki/Liouville%27s_theorem_(complex_analysis))